waves (lec 01)

Ref:- Schaum's outline of "Theory and Problems of electromagnetics" 2nd edition, Joseph A. edminister.

Contents:-

- ch(1):- Vector Analysis & coordinate systems.
- ch(2): Coloumb Forces & Electric Field intensity
- ch(3):- Electric Flux & gauss' law
- × ch(4):- Divergence & Divergence Theorem × ده أهم شغل عليك (curl, Div, grad)
- ch(5):- Electrostatic Field, work, Energy, Potential
- ch(6):- Current & Current Density & conductors.
- ch(7):- Capacitance & Dielectric material

Time Table مبدئي

ch(1) + sheet(2)	٢٠/٦	الثلاثاء	ch(1) + sheet 1	١٧/٦	السبت
اجازة العيد	٢٧/٦	الثلاثاء	ch(2) ~~Sheet~~ (3) Curl - Div - grade	٢٤/٦ (3,4,5 shoubra)	السبت
ch(2) sheet(4) → 6 shoubra	٤/٧	الثلاثاء	ch(2) ~~Sheet~~ ge	١/٧	السبت
Midterm + sheet ch(3)	١١/٧	Midterm الثلاثاء	ch(3) ~~sheet~~ Flux	٨/٧	السبت
ch(4) sheet	١٨/٧	الثلاثاء	ch(4) work energy	١٥/٧	السبت
ch(5) sheet	٢٥/٧	الثلاثاء	ch(5) current Density	٢٢/٧	السبت
ch(6) sheet	١/٨	الثلاثاء	ch(6) capacitance	٢٩/٧	السبت

final

(2)

chapter (1)
Vector Analysis

* Scalar Quantity :- Quantity have no directional information & has value only (Temp, Mass, ...)
(Field)

* Vector Quantity :- Quatity have both direction & value (velocity, force, displacement,)
(Field)

* field :- Distribution of any Quantity in space

* Electric Field occurred due to
  → Static charge → Point charge / → line " / → Surface " / → volume
  → potential difference
  → Magnetic field → due to Magnet / → E.f / → Curring pass in Conductor

vector $\bar{A}$

$$\bar{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$$

A_x → مقدار ال vector في اتجاه x

$\hat{a_x}, \hat{a_y}, \hat{a_z}$ → unit vector

$|\hat{a_x}| = |\hat{a_y}| = |\hat{a_z}| = 1$

~~الـعلاقة بيكون لها نفس تعويض شكل~~

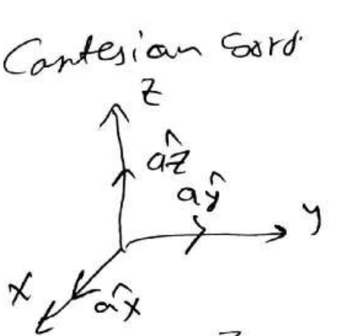
النقطة P احداثياتها x, y, z

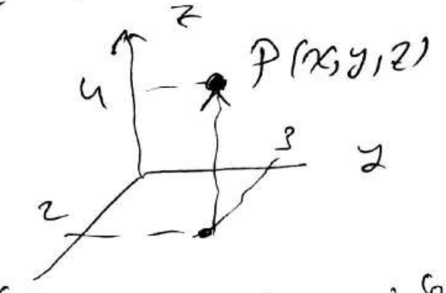
→ $P(2,3,4)$

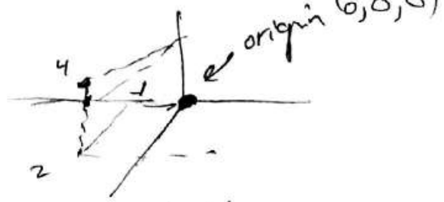
$\therefore \bar{A} = 2\hat{a_x} + 3\hat{a_y} + 4\hat{a_z}$

$|\bar{A}| = \sqrt{2^2+3^2+4^2} = \sqrt{29}$

Cartesian coord







(2,-1,4)

Planes { xy , z const / xz , y const / yz , x const

$\bar{A}+\bar{B} = \bar{B}+\bar{A}$ Commutative

$(\bar{A}+\bar{B})+\bar{C} = \bar{A}+(\bar{B}+\bar{C})$ Associative

$\bar{A}-\bar{B} = \bar{A}+(-\bar{B})$.

Vector (directed from point to point)

$P_1 \; (x_1, y_1, z_1) \longrightarrow P_2 \; (x_2, y_2, z_2)$

$$\bar{R} = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

$$|\bar{R}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\hat{a}_R = \frac{\bar{R}}{|\bar{R}|}$$

unit vector in direction of R

ex $\bar{A} = 2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z$

$\hat{a}_A = \frac{2}{\sqrt{20}}\hat{a}_x + \frac{3}{\sqrt{20}}\hat{a}_y + \frac{4}{\sqrt{20}}\hat{a}_z$

Product of vectors < dot Scalar, cross ~~~~

(1) $\bar{A} + \bar{B} = (A_x + B_x)\hat{a}_x + (A_y + B_y)\hat{a}_y + (A_z + B_z)\hat{a}_z$

for $A = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$B = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$

(2) Scalar (dot) = product of 2 vectors ← ناتج الضرب قياسي

$\bar{A} \cdot \bar{B} = |A||B| \cos\theta_{AB}$

if $\bar{A} \perp \bar{B}$, $\theta = 90$

$\therefore \bar{A} \cdot \bar{B} = |A||B| \cos 90 = \text{zero}$

& $|a_x||a_x| = 1$

$|a_y||a_y| = 1$

$|a_z||a_z| = 1$

$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$

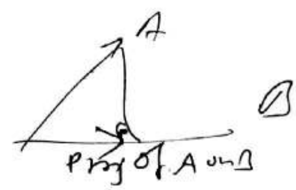
$\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$

if $\bar{A} \cdot \bar{B} = 0 \therefore \bar{A} \perp \bar{B}$

$\hat{a}_x \cdot \hat{a}_x = \hat{a}_y \cdot \hat{a}_y = \hat{a}_z \cdot \hat{a}_z = 1$

$\hat{a}_x \cdot \hat{a}_y = \hat{a}_y \cdot \hat{a}_z = \hat{a}_x \cdot \hat{a}_z = \text{zero}$

$$\cos\theta_{AB} = \frac{\bar{A} \cdot \bar{B}}{|A||B|}$$



Projection of $\bar{A}$ on $\bar{B} = |\bar{A}| \cos\theta_{AB} = |\bar{A}| \cos\theta_{AB} \frac{|\bar{B}|}{|\bar{B}|}$

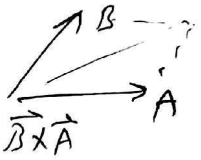
$$= \frac{\bar{A} \cdot \bar{B}}{|\bar{B}|} = \bar{A} \cdot \hat{a}_B$$

(a)

2 - Cross product (vector product)

ناتج متجه

1. $\bar{A} \times \bar{B} = |A||B| \sin\theta \, \bar{a}_n \rightarrow$ وحدة متجه عمودي ع A ، B



2. $\bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$

باستخدام قاعدة اليد اليمنى RH Rule كالتقول $\bar{A} \times \bar{B}$ ∴ $\bar{A}$ الأول مفرودة ثم B وتكون الطالع العمودي على تكون موجبة إما لو $\bar{A} \times \bar{B}$ الطالع عمودي لتحت أي سالبة

3. If $A \parallel B$ (parallel يوازي) $\therefore \theta = 0$, $\bar{A} \times \bar{B} = 0$

4. $\bar{a}_x \times \bar{a}_x = \bar{a}_y \times \bar{a}_y = \bar{a}_z \times \bar{a}_z = 0$

- $\hat{a}_x \times \hat{a}_y = \hat{a}_z$
- $\hat{a}_x \times \hat{a}_z = -\hat{a}_y$
- $\hat{a}_y \times \hat{a}_x = -\hat{a}_x$
- $\hat{a}_y \times \hat{a}_z = \hat{a}_x$
- $\hat{a}_z \times \hat{a}_x = \hat{a}_y$
- $\hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$$a_n = \frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|} \rightarrow |A||B| \sin\theta$$

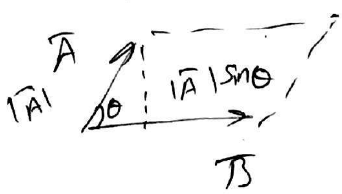
$$\bar{A} \cdot (\bar{B} \times \bar{C}) = (\bar{A} \times \bar{B}) \cdot \bar{C}$$

5. $\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

لحفظها

6. $\bar{A} \times (\bar{B} + \bar{C}) = \bar{A} \times \bar{B} + \bar{A} \times \bar{C} \neq (\bar{A} \times \bar{B}) \times C$

7. Note $|\bar{A} \times \bar{B}|$ = area of parallelogram whose sides are $\bar{A}$ & $\bar{B}$ = 2 area of Triangle



area = length of Base × hight

$|\bar{A} \times \bar{B}| = |A||B| \sin\theta_{AB}$

$$a_n = \frac{|\bar{A} \times \bar{B}|}{\cancel{|A|}} \qquad \frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|}$$

Sheet (1)

Q1) Find the vector $\bar{A}$ directed from $(2, -4, 1)$ to $(0, -2, 0)$ in Cartesian coordinates & find unit vector along $\bar{A}$.

Sol/

$$(2, -4, 1) \xrightarrow{\bar{A}} (0, -2, 0)$$

$$\bar{A} = (0-2)\hat{a_x} + (-2-(-4))\hat{a_y} + (0-1)\hat{a_z}$$

$$\bar{A} = -2\hat{a_x} + 2\hat{a_y} - \hat{a_z}$$

$$\hat{a}_A = \frac{\bar{A}}{|\bar{A}|} = \frac{-2\hat{a_x} + 2\hat{a_y} - 2\hat{a_z}}{\sqrt{(-2)^2 + (2)^2 + (-1)^2}} = -\frac{2}{3}\hat{a_x} + \frac{2}{3}\hat{a_y} - \frac{1}{8}\hat{a_z}$$

Q2) Show that $\bar{A} = 4\hat{a_x} - 2\hat{a_y} - \hat{a_z}$ & $\bar{B} = \hat{a_x} + 4\hat{a_y} - 4\hat{a_z}$ are Perpendicular متعامدين

if $\bar{A} \cdot \bar{B} = 0 \quad \therefore A \perp B$

$$\bar{A} \cdot \bar{B} = |\bar{A}|\,|\bar{B}| \cos\theta_{AB} = 0 \longrightarrow \theta_{AB} = 90^\circ$$

Now $\bar{A} \cdot \bar{B} = (4)(1) + (-2)(4) + (-1)(-4) = 4 - 8 + 4 = 0$

$\therefore \bar{A} \cdot \bar{B} = 0 \quad \therefore \bar{A} \perp \bar{B}$

Q3) Determine the Smallest angle between $\bar{A} = 2\hat{a_x} + 4\hat{a_y}$ & $\bar{B} = 6\hat{a_y} - 4\hat{a_z}$ using cross Product and also dot product

Sol

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$\boxed{1}$ using cross product

$$\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\theta_{AB} \, \hat{a}_n$$

بيطلع هون قيمة موجبة عشان أخذنا مطلق ال $|\bar{A} \times \bar{B}|$

$$|\bar{A} \times \bar{B}| = |\bar{A}||\bar{B}| \sin\theta_{AB}$$

$$\therefore \sin\theta_{AB} = \frac{|\bar{A} \times \bar{B}|}{|\bar{A}||\bar{B}|}$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & 4 & 0 \\ 0 & 6 & -4 \end{vmatrix} = -16\hat{a}_x - (-8)\hat{a}_y + 12\hat{a}_z$$

$$\therefore \bar{A} \times \bar{B} = -16\hat{a}_x + 8\hat{a}_y + 12\hat{a}_z$$

$$|\bar{A} \times \bar{B}| = \sqrt{(-16)^2 + (8)^2 + (12)^2} = \sqrt{464}$$

$$|\bar{A}| = \sqrt{(2)^2 + (4)^2} = \sqrt{20}$$

$$|\bar{B}| = \sqrt{(6)^2 + (-4)^2} = \sqrt{52}$$

$$\sin\theta_{AB} = \sin^{-1} \frac{\sqrt{464}}{\sqrt{20}\sqrt{52}} = \sin^{-1} 0.6679 \approx 41.90^\circ \text{ or } 138.10^\circ$$

$\boxed{2}$ using dot product

$$\bar{A} \cdot \bar{B} = |\bar{A}||\bar{B}| \cos\theta_{AB}$$

$$\bar{A} \cdot \bar{B} = (2\hat{a}_x + 4\hat{a}_y + 0\hat{a}_z) \cdot (0\hat{a}_x + 6\hat{a}_y - 4\hat{a}_z)$$

$$= 2 \times 0 + 4 \times 6 + 0 \times -4 = 24$$

$$\therefore \cos\theta_{AB} = \frac{\bar{A} \cdot \bar{B}}{|\bar{A}| \cdot |\bar{B}|} = \frac{24}{\sqrt{20}\sqrt{52}} = 0.7442$$

$$\therefore \theta = \cos^{-1} 0.7442 \approx 41.9^\circ$$

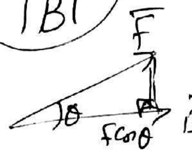
/ Given $\bar{F} = (y-1)\hat{a}_x + 2x\,\hat{a}_y$, find the vector at $(2,2,1)$ and its Projection on $\bar{B} = 5\bar{a}_y - \bar{a}_y + 2\bar{a}_z$

Sol /

at point $(2, 2, 1)$,
$x \quad y \quad z$

$$\boxed{\bar{F} = (2-1)\hat{a}_x + 2\times 2\,\hat{a}_y = \hat{a}_x + 4\hat{a}_y}$$

$$\text{Projection of } \bar{F} \text{ on } \bar{B} = \frac{\bar{F}\cdot\bar{B}}{|\bar{B}|} = \bar{F}\cdot\hat{a}_B = |\bar{F}|\cos\theta$$



جد مهم
أهم القانون

$$\text{Projection} = |\bar{F}|\cos\theta$$

$$\bar{F}\cdot\bar{B} = \underbrace{|\bar{F}|\,|\bar{B}|\cos\theta} \qquad \therefore\ |\bar{F}|\cos\theta = \frac{\bar{F}\cdot\bar{B}}{|\bar{B}|}$$

$$\therefore\ \bar{F}\cdot\bar{B} = 5\times 1 + 4\times(-1) + 0\times 2 = 1$$

$$|\bar{B}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{30}$$

$$\therefore\ \text{projection} = \frac{1}{\sqrt{30}}$$

Q5) If $\bar{A} = \hat{a}_x + 2\hat{a}_y - 3\hat{a}_z$ and $\bar{B} = 2\hat{a}_x - \hat{a}_y + \hat{a}_z$

determine:-
a) The magnitude of projection of $\bar{B}$ on $\bar{A}$
b) The smallest angle between $\bar{A}$ and $\bar{B}$
c) the vector Projection $\bar{A}$ onto $\bar{B}$
d) A unit vector perpendicular to plane Containing $\bar{A}$ & $\bar{B}$

Sol / (a) Proj of $\bar{B}$ on $\bar{A}$ $= \dfrac{\bar{B}\cdot\bar{A}}{|\bar{A}|} = \dfrac{2\times1 - 1\times2 - 3\times1 = -3}{\sqrt{1^2+2^2+(-3)^2}} = \dfrac{-3}{\sqrt{1}}$

magnitude of proj $= \left|\dfrac{-3}{\sqrt{14}}\right| = \dfrac{+3}{\sqrt{14}}$ ✱

b) Smallest angle $\theta_{AB} = \cos^{-1}\dfrac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|} = \cos^{-1}\dfrac{-3}{\sqrt{14}\,\sqrt{6}} \Rightarrow 109.1^\circ$

c) Proj $\bar{A}$ onto $\bar{B}$ $= \dfrac{\bar{A}\cdot\bar{B}}{|\bar{B}|} = \dfrac{-3}{\sqrt{6}}$

d) Unit Vector ~~Parallel~~ perpendicular to ~~A~~ plane contain $\bar{A}$, $\bar{B}$

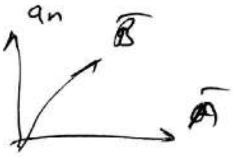
دو سيدي مع $\bar{A} \times \bar{B}$ ← $\hat{a}_n$ متجه وحدة عمودي الكل

$$\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\theta_{AB}\, \hat{a}_n$$

$$\hat{a}_n = \frac{\bar{A} \times \bar{B}}{|\bar{A}||\bar{B}| \sin\theta_{AB}} \equiv \frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|}$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 2 & -3 \\ 2 & -1 & 1 \end{vmatrix} = -\hat{a}_x - 7\hat{a}_y - 5\hat{a}_z$$

$$|\bar{A} \times \bar{B}| = \sqrt{(-1)^2 + (-7)^2 + (-5)^2} = \sqrt{75}$$



$$\therefore \hat{a}_n = -\frac{1}{\sqrt{75}}\hat{a}_x - \frac{7}{\sqrt{75}}\hat{a}_y - \frac{5}{\sqrt{75}}\hat{a}_z$$

مكمل التعريف: $\theta = 109.1^\circ$

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6 given $\bar{A} = \hat{a}_x + \hat{a}_y$, $\bar{B} = \hat{a}_x + 2\hat{a}_z$, $\bar{C} = 2\hat{a}_y + \hat{a}$

find $(\bar{A} \times \bar{B}) \times \bar{C}$ & copare it with $\bar{A} \times (\bar{B} \times \bar{C})$

Sol $(\bar{A} \times \bar{B}) \times C$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{vmatrix}$$

$$\bar{A} \times \bar{B} = 2\hat{a}_x - 2\hat{a}_y - \hat{a}_z$$

$$(\bar{A} \times \bar{B}) \times \bar{C} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & -2 & -1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= -2\hat{a}_y + 4\hat{a}_z$$

$\bar{A} \times (\bar{B} \times \bar{C})$

$$\bar{B} \times \bar{C} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= -4\hat{a}_x - \hat{a}_y + 2\hat{a}_z$$

$$\bar{A} \times (\bar{B} \times \bar{C}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 1 & 0 \\ -4 & -1 & 2 \end{vmatrix}$$

$$= 2\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$$

$$\therefore \bar{A} \times (\bar{B} \times \bar{C}) \neq (\bar{A} \times \bar{B}) \times \bar{C}$$

7) For previous problem Find $\bar{A}\cdot\bar{B}\times\bar{C}$

& Compare it with $\bar{A}\times\bar{B}\cdot\bar{C}$

(~~Repeat~~)

hint

مدام مفيش قوس يبقى دائماً اعمل X الاول

$\rightarrow \bar{A}\cdot(\bar{B}\times\bar{C})$

$\because \bar{B}\times\bar{C} = -4\hat{a}_x - \hat{a}_y + 2\hat{a}_z$ من السابق

$\bar{A}\cdot\bar{B}\times\bar{C} = (1\times -4) + (1\times -1) + (0\times 2) = -5$

$\rightarrow \bar{A}\times\bar{B}\cdot C \longrightarrow (\bar{A}\times\bar{B})\cdot C$

$\Rightarrow \bar{A}\times\bar{B} = 2\hat{a}_x - 2\hat{a}_y - \hat{a}_z$

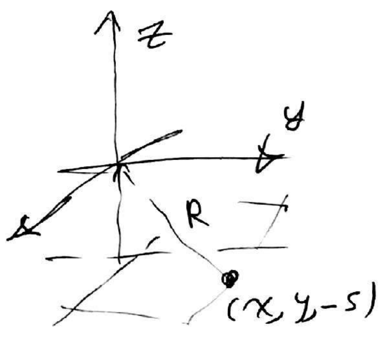
$(\bar{A}\times\bar{B})\cdot\bar{C} = (2\times 0) + (-2\times 2) + (-1\times 1) = -5$

$$\boxed{\bar{A}\cdot(\bar{B}\times\bar{C}) = (\bar{A}\times\bar{B})\cdot C}$$

Q8) Express the Unit vector which directed toward the origin from arbitary point on the plane $z=-5$

الحل

$P(x,y,-5)$ احداثيات النقطة



$\bar{R} = \text{Origin} \xleftrightarrow{\text{towards}} P$

$= (-x+0)\hat{a}_x + (-y+0)\hat{a}_y + (0-(-5))\hat{a}_z$

$\bar{R} = -x\hat{a}_x - y\hat{a}_y + 5\hat{a}_z$

$|\bar{R}| = \sqrt{x^2+y^2+25}$

$$\hat{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-x\hat{a}_x - y\hat{a}_y + 5\hat{a}_z}{\sqrt{x^2+y^2+25}}$$

Report

9. Given 2 vectors

$\bar{A} = -\bar{a}_x - 3\bar{a}_y - 4\bar{a}_z$

$\bar{B} = 2\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z$

& Point $C(1,3,4)$

find (1) $\bar{R}_{AB}$ (2) $|\bar{A}|$ (3) $\bar{a}_A$ (4) $\bar{a}_{AB}$

(5) unit vector directed from C toward A

Sol

(1) $\bar{R}_{AB} = \bar{B} - \bar{A} = (2-(-1))\hat{a}_x + (2-(-3))\hat{a}_y + (2-(-4))\hat{a}_z$

$= 3\hat{a}_x + 5\hat{a}_y + 6\hat{a}_z$

(2) $|\bar{A}| = \sqrt{(-1)^2 + (-3)^2 + (-4)^2} = 5.099$

(3) $\bar{a}_A = \dfrac{\bar{A}}{|\bar{A}|} = \dfrac{-1}{5.099}\hat{a}_x - \dfrac{3}{5.099}\hat{a}_y - \dfrac{4}{5.099}\hat{a}_z$

(4) $\bar{a}_{AB} = \dfrac{\bar{R}_{AB}}{|\bar{R}_{AB}|} = \dfrac{3\hat{a}_x + 5\hat{a}_y + 6\hat{a}_z}{\sqrt{3^2 + 5^2 + 6^2} = \sqrt{70}}$

(5) $\bar{a}_{CA} = \dfrac{\bar{R}_{CA}}{|\bar{R}_{CA}|} = \dfrac{\bar{A} - \bar{C}}{|\bar{R}_{CA}|} = \dfrac{(-1-1)\hat{a}_x - 3 - 3\hat{a}_y - 4 - 4\hat{a}_z}{\sqrt{(-2)^2 + (-6)^2 + (-8)^2}}$

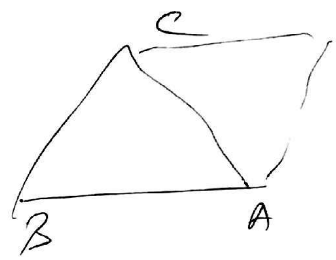
$= \dfrac{-2\hat{a}_x - 6\hat{a}_y - 8\hat{a}_z}{\sqrt{104}}$

(10) triangle defined by $A\,(2, -5, 1)$
$B\,(-3, 2, 4)$
$C = (0, 3, 1)$

Report

find ① $\bar{R}_{BC} \times \bar{R}_{BA}$
② area of triangle
③ unit vector perpendicular to plane of triangle

Sol



$$\bar{R}_{BC} \overset{C-B}{=} (0+3)\hat{a}_x + (3-2)\hat{a}_y + (1-4)\hat{a}_z$$
$$= 3\hat{a}_x + \hat{a}_y - 3\hat{a}_z$$

$$\bar{R}_{BA} \overset{A-B}{=} 5\hat{a}_x - 7\hat{a}_y - 3\hat{a}_z$$

$$\bar{R}_{BC} \times \bar{R}_{BA} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 3 & 1 & -3 \\ 5 & -7 & -3 \end{vmatrix}$$
$$= -24\hat{a}_x - 6\hat{a}_y - 26\hat{a}_z$$

② area of triangle $= \frac{1}{2}\,|\bar{R}_{BC} \times \bar{R}_{BA}| = \frac{1}{2}\sqrt{(-24)^2 + (-6)^2 + (-26)^2}$
$= 17.944$ square unit

$\frac{1}{2}$ مساحة متوازي الاضلاع المكون من محصلة
$BC \times BA$

③ unit vector $\perp \triangle \Rightarrow \hat{a}_n$

$$\bar{R}_{BC} \times \bar{R}_{BA} = |\bar{R}_{BC} \times \bar{R}_{BA}|\,\bar{a}_n$$

$$\therefore\ \bar{a}_n = \frac{-24\hat{a}_x - 6\hat{a}_y - 26\hat{a}_z}{\sqrt{(-24)^2 + (-6)^2 + (-26)^2}} = -\frac{12\hat{a}_x + 3\hat{a}_y + 13\hat{a}_z}{\sqrt{1288}}$$